

PROGNOSETAFEL AG 2020

Modelspecificatie

Toelichting



Opzet Toelichting:

- **AG2018:**
 - Modelstructuur AG2020 in de basis gelijk aan AG2018.
- **AG2018 → AG2020:**
 - Vragen die geleid hebben tot wijzigingen.
- **AG2020**
- **AG2020 ↔ AG2018:**
 - AG2020 in vergelijking met AG2018.

The background features a light blue line graph on a white grid. The x-axis represents years from 1928 to 2004, with labels every two years. The y-axis represents values, with labels from 0 to 2000 in increments of 20. A magnifying glass is positioned over the graph, focusing on the data points between 1970 and 1990. The text is centered over the graph.

Modelstructuur AG2020 in de basis gelijk aan AG2018

AG2018

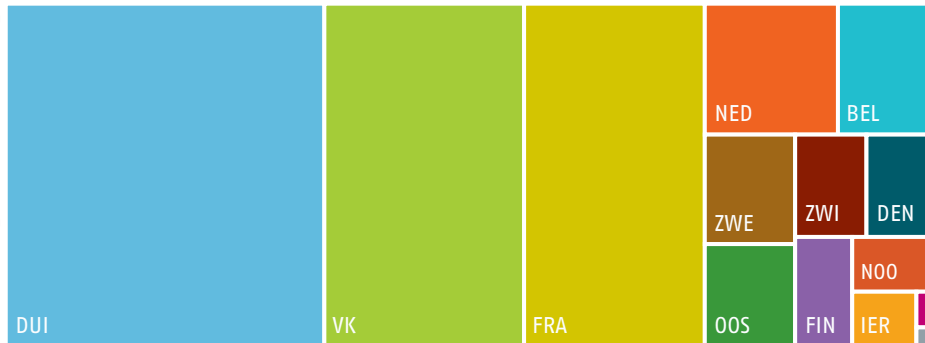


AG2018-Model

Startpunt éénjaarsoverlijdenskansen Nederlandse populatie: $q_x^g(t) = 1 - e^{-\mu_x^g(t)}$.

Beschikbare data $D_{x,t}^{g,EU}$ & $E_{x,t}^{g,EU}$, $t = 1970, \dots, 2016$, en $D_{x,t}^{g,NL}$ & $E_{x,t}^{g,NL}$, $t = 1970, \dots, 2017$, $x = 0, 1, \dots, 90$.

■ OOS ■ BEL ■ DEN ■ FIN ■ FRA ■ DUI ■ IJS
■ IER ■ LUX ■ NED ■ NOO ■ ZWE ■ ZWI ■ VK



Opsplitsing $\ln(\mu_x^g(t))$:

Europa: $\ln(\mu_x^{g,EU}(t)) = A_x^g + B_x^g K_t^g,$

NL-afwijking: $\ln(\mu_x^{g,NL}(t)) = \alpha_x^g + \beta_x^g \kappa_t^g$ +

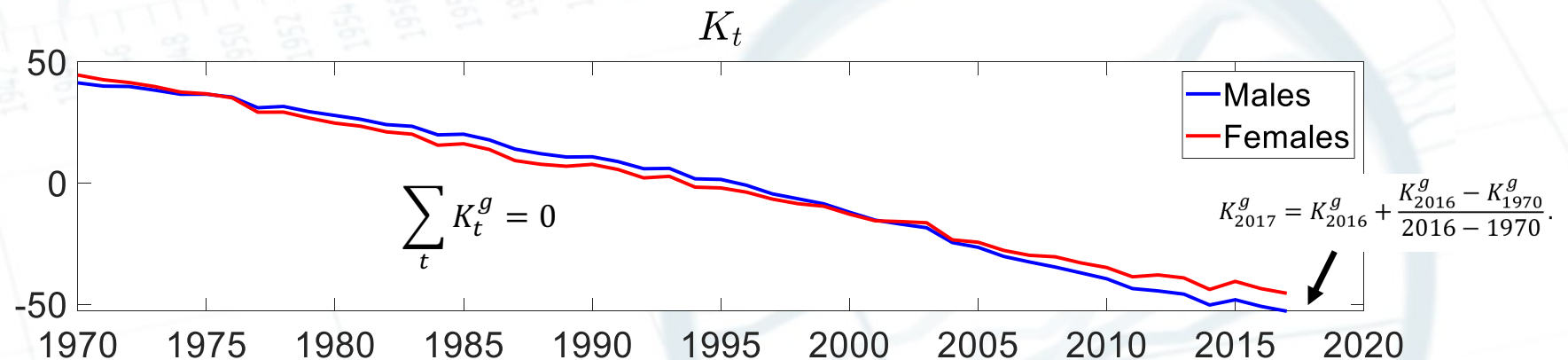
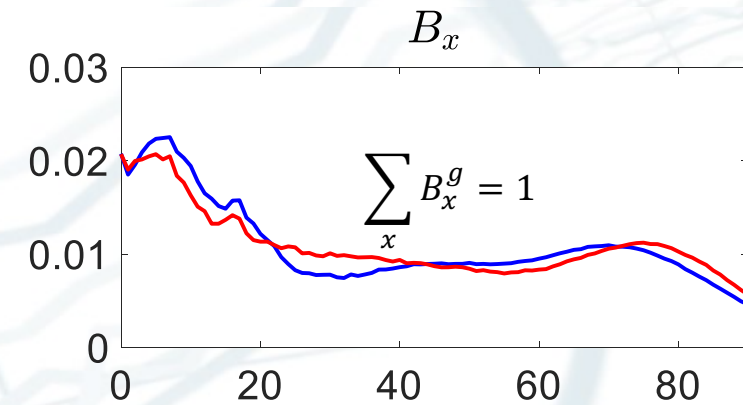
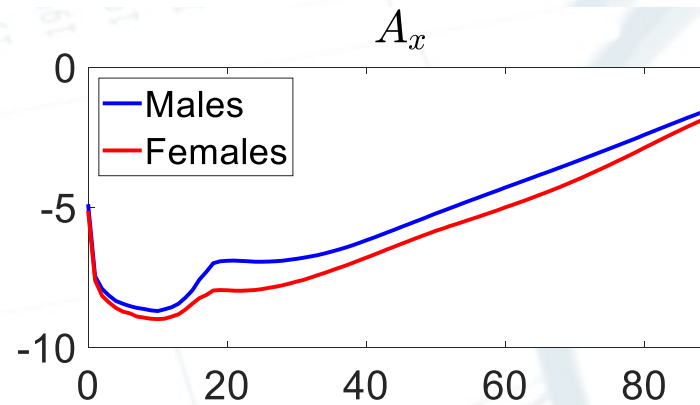
NL: $\ln(\mu_x^g(t)) = \ln(\mu_x^{g,EU}(t)) + \ln(\mu_x^{g,NL}(t))$

AG2018-Model: Kalibratie Europa

Verdelingsveronderstelling: $D_{x,t}^{g,EU} | E_{x,t}^{g,EU} \sim \text{Pois}(\mu_x^{g,EU}(t) E_{x,t}^{g,EU})$.

$$\max_{\{A_x^g, B_x^g, K_t^g\}} \prod_{x \in X^0} \prod_{t \in T^0} \frac{(E_{x,t}^{g,EU} \mu_x^{g,EU}(t))^{D_{x,t}^{g,EU}} \exp(-E_{x,t}^{g,EU} \mu_x^{g,EU}(t))}{D_{x,t}^{g,EU}!}$$

$$\mu_x^{g,EU}(t) = e^{A_x^g + B_x^g K_t^g}$$



$X^0 = \{0, 1, 2, \dots, 90\}$
 $T^0 = \{1970, \dots, 2016\}$



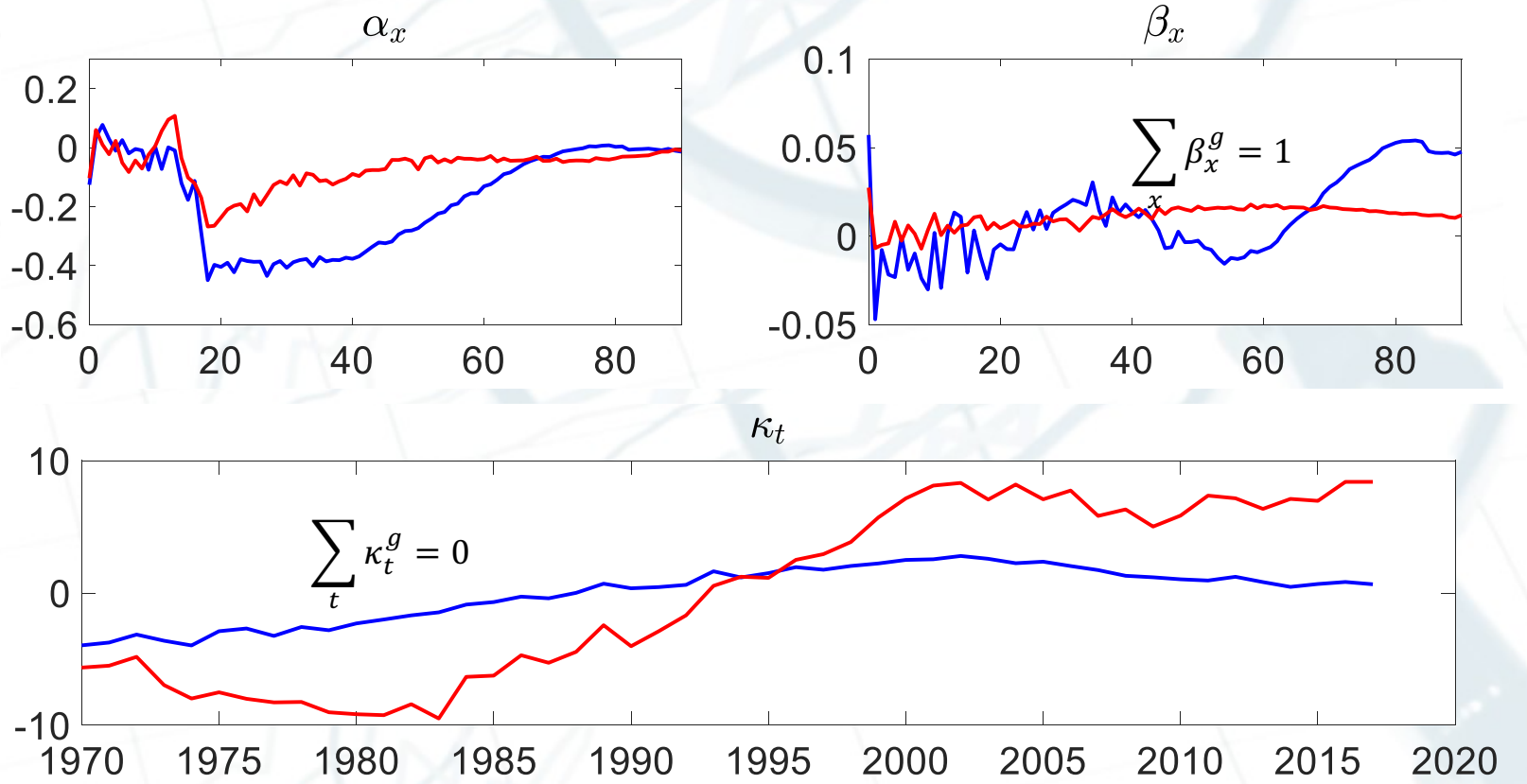
AG2018-Model: Kalibratie NL-afwijking

Verdelingsveronderstelling: $D_{x,t}^{g,NL} | E_{x,t}^{g,NL} \sim \text{Pois}(\mu_x^g(t) E_{x,t}^{g,NL})$.

$$\max_{\{\alpha_x^g, \beta_x^g, \kappa_t^g\}} \prod_{x \in X^0} \prod_{t \in T^*} \frac{(E_{x,t}^{g,NL} \mu_x^g(t))^{D_{x,t}^{g,NL}} \exp(-E_{x,t}^{g,NL} \mu_x^g(t))}{D_{x,t}^{g,NL}!}$$

$$\mu_x^g(t) = \mu_x^{g,EU}(t) \mu_x^{g,NL}(t):$$

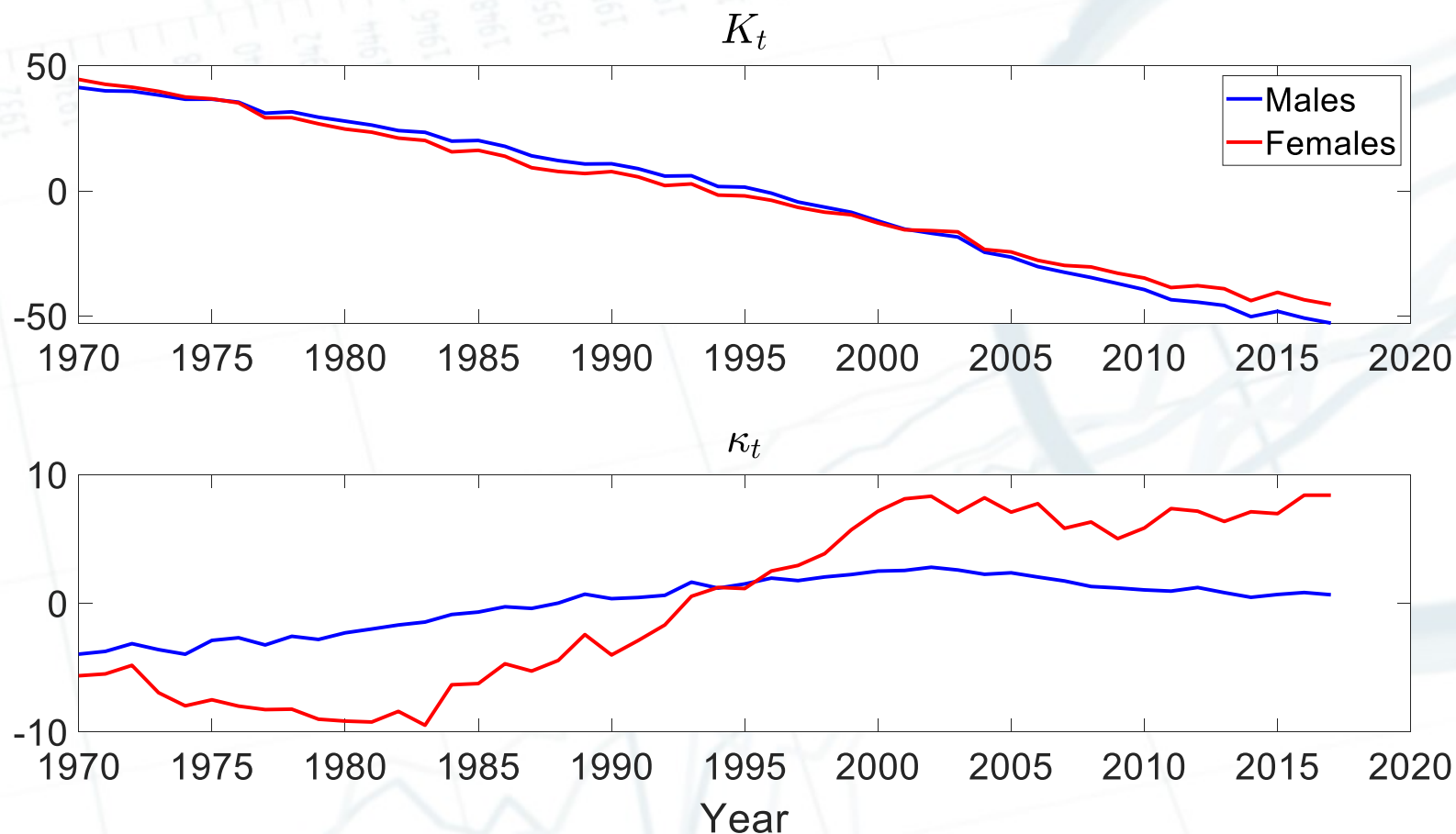
- $\mu_x^{g,EU}(t) \rightarrow \hat{\mu}_x^{g,EU}(t) = e^{\hat{A}_x^g + \hat{B}_x^g \hat{K}_t^g}$
- $\mu_x^{g,NL}(t) = e^{\alpha_x^g + \beta_x^g \kappa_t^g}$



$X^0 = \{0,1,2, \dots, 90\},$
 $T^* = \{1970, \dots, 2017\}$



AG2018-Model: Kalibratie tijdstrends



Modelveronderstelling en Schattingsresultaten:

$$K_t^M = K_{t-1}^M + \theta^M + \epsilon_t^M, \text{ met } \hat{\theta}^M \approx -2.04$$

$$K_t^F = K_{t-1}^F + \theta^F + \epsilon_t^F, \text{ met } \hat{\theta}^F \approx -1.96$$

$$\kappa_t^M = a^M \kappa_{t-1}^M + \delta_t^M, \text{ met } \hat{a}^M \approx 0.98$$

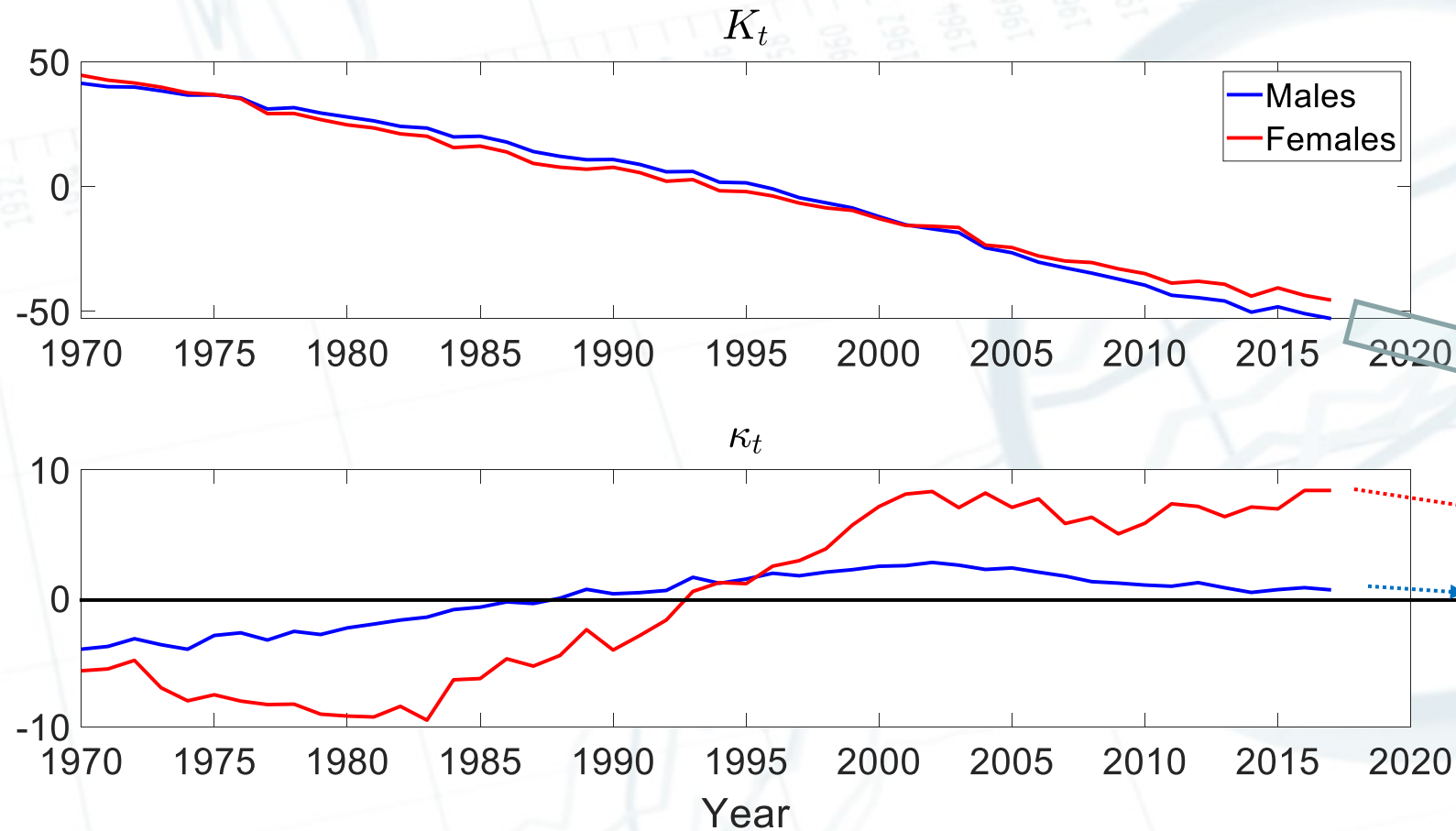
$$\kappa_t^F = a^F \kappa_{t-1}^F + \delta_t^F, \text{ met } \hat{a}^F \approx 0.99$$

$$(\epsilon_t^M, \delta_t^M, \epsilon_t^F, \delta_t^F) \sim i.i.d. N(0, C)$$

$t = 1971, \dots, 2017$



AG2018-Model: Projecties



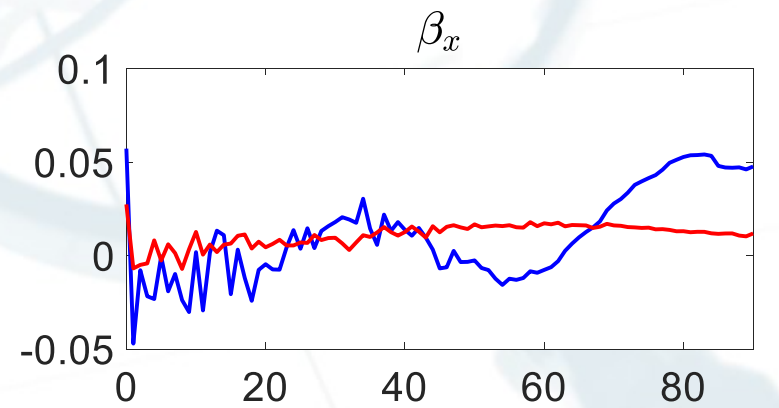
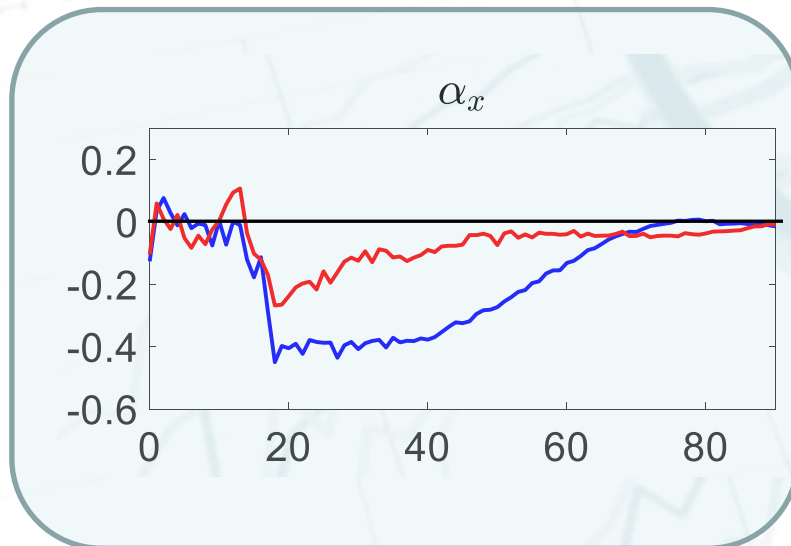
“Best estimates”
 K_{T+t}^g & κ_{T+t}^g

$$\begin{aligned}
 q_x^g(T+t) &= 1 - e^{-\mu_x^g(T+t)} \text{ met} \\
 &\ln(\mu_x^g(T+t)) \\
 &= \ln(\mu_x^{g,EU}(T+t)) + \ln(\mu_x^{g,NL}(T+t)) \\
 &= A_x^g + B_x^g K_{T+t}^g + \alpha_x^g + \beta_x^g \kappa_{T+t}^g
 \end{aligned}$$



AG2018-Model: Afwijking NL-EU

- Afwijking is $\alpha_x^g + \beta_x^g \kappa_{T+\tau}^g \rightarrow_{\tau \rightarrow \infty} \alpha_x^g = \frac{1}{T} \sum_t \ln(\mu_x^{g,NL}(t)) \neq 0$. ($\frac{1}{T} \sum_t \ln(\mu_x^{g,NL}(t)) =$ steekproefgemiddelde)



- Afwijking speelt steeds minder belangrijke rol vanwege steeds verder dalende Europese trend.
- Levensverwachtingen NL en EU (zeker op langere termijn) steeds dichterbij elkaar.

$$q_x^g(t) = 1 - e^{-\mu_x^g(t)}$$

Sluiting Prognosetafel

- Leeftijdsafhankelijke parameters voor leeftijden 0 – 90.
- Voor leeftijden 91 – 120 wordt **Kannisto's sluitingsmethode** gebruikt.

Voor leeftijden $x \in \{91, \dots, 120\}$, volgt $\mu_x^g(t)$ uit

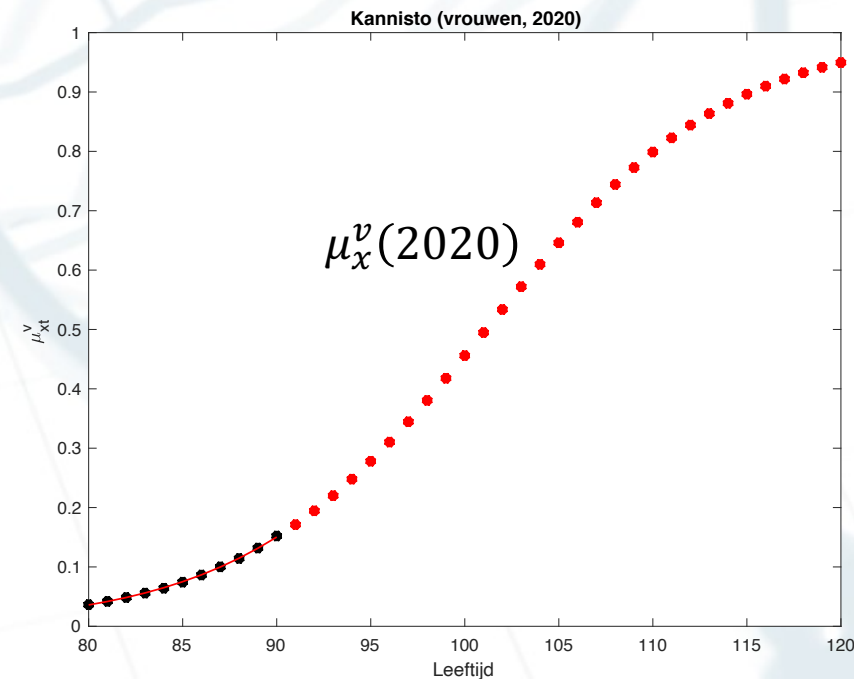
- $$\mu_x^g(t) = L\left(\sum_{k=80}^{90} w_k(x) L^{-1}\left(\mu_k^g(t)\right)\right),$$

met

- $$L(x) = \frac{1}{1+e^{-x}},$$

- $$L^{-1}(x) = -\ln\left(\frac{1}{x}-1\right) = \ln(x) - \ln(1-x),$$

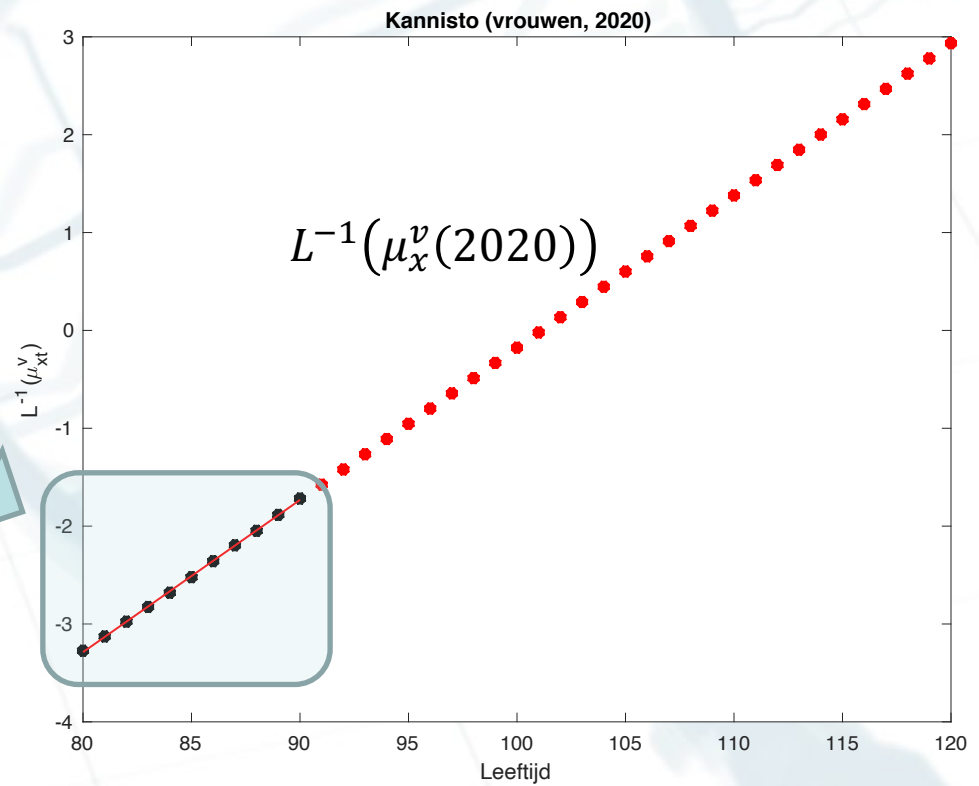
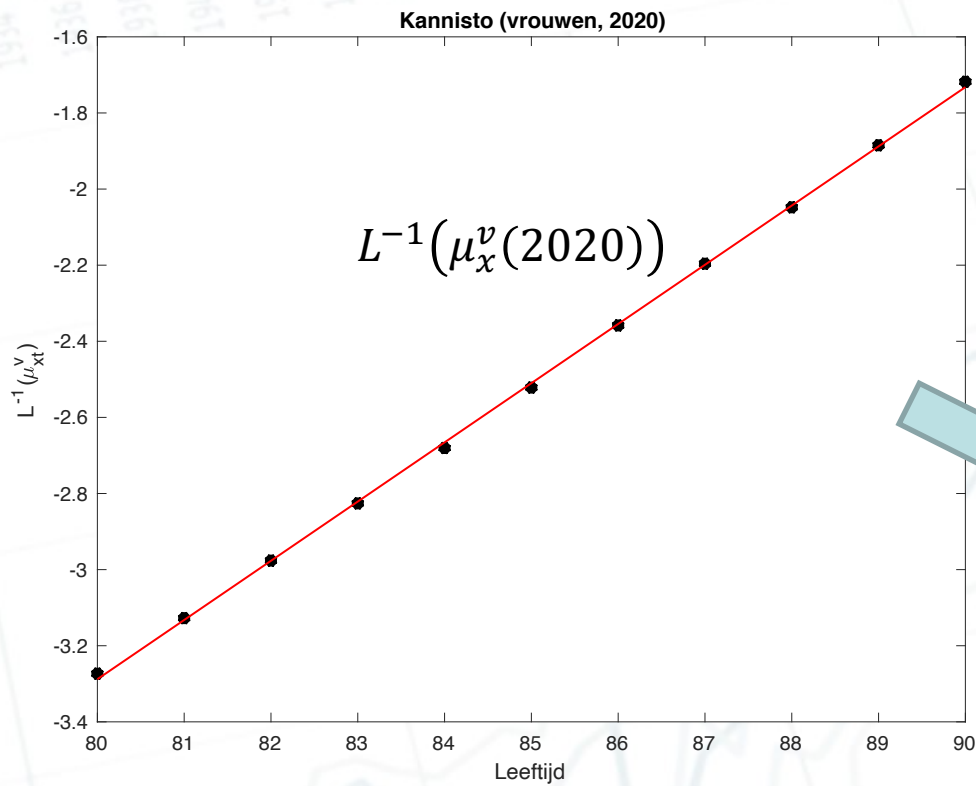
- $$w_k(x) = \frac{1}{11} + \frac{(k-85)(x-85)}{110}.$$



$$q_x^g(t) = 1 - e^{-\mu_x^g(t)}$$

Sluiting Prognosetafel

$$L^{-1}(\mu_x^v(2020)) = \ln\left(\frac{\mu_x^v(2020)}{1 - \mu_x^v(2020)}\right) = -15.7339 + 0.1556x + \text{error}$$



The background features a light blue line chart with multiple data series plotted against a grid. The x-axis is labeled with years from 1928 to 2004 in two-year increments. A large, semi-transparent magnifying glass is positioned over the chart, focusing on the data lines. The overall aesthetic is clean and professional, typical of a financial or actuarial presentation.

Vragen die geleid hebben tot wijzigingen

AG2018 → AG2020

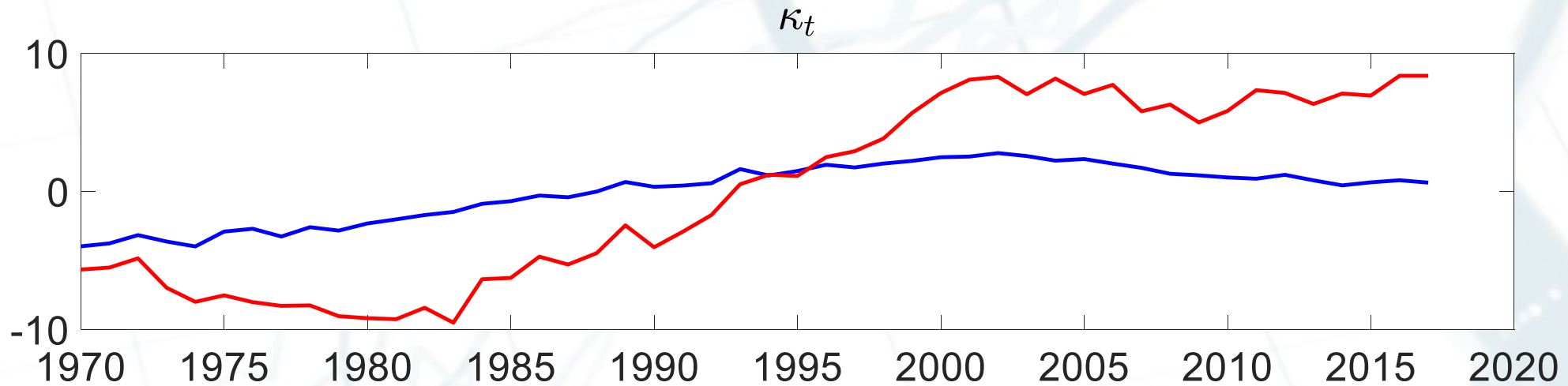


Vragen die geleid hebben tot wijzigingen

- Convergeren de tijdsreeksen (κ_t^g) voor het verschil tussen Nederland en de andere Europese landen in verwachting naar nul?
- Hoe lang moet de historie zijn die meegenomen wordt bij het maken van de prognose?

Convergentie κ_t^g

Convergeren de tijdsreeksen (κ_t^g) voor het verschil tussen Nederland en de andere Europese landen in verwachting naar nul?



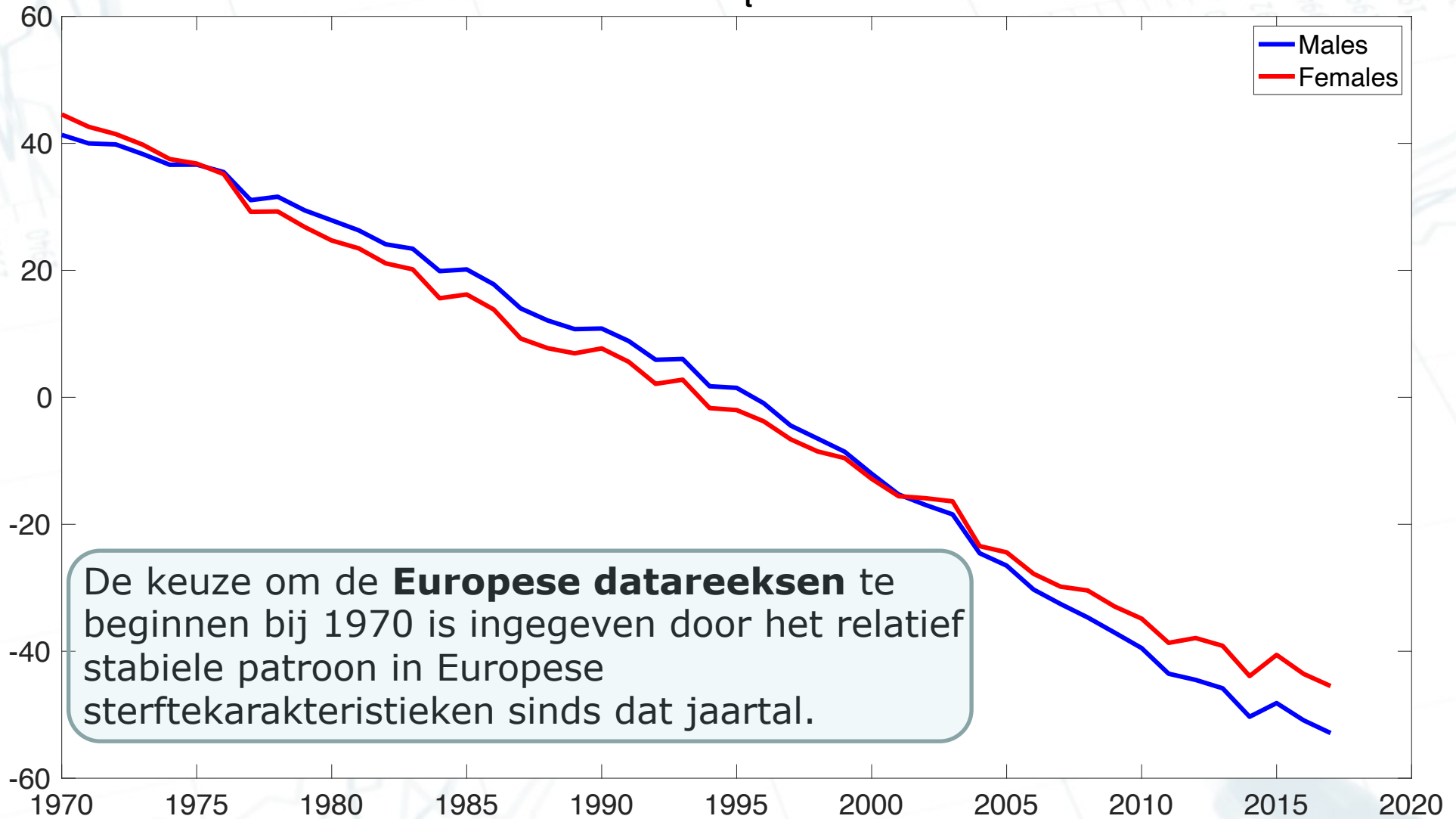
Algemene AR(1)-specificatie (met **constante**) : $\kappa_t^g = a^g \kappa_{t-1}^g + c^g + \delta_t^g$
Er geldt dan $E(\kappa_t^g) \rightarrow_{t \rightarrow \infty} \frac{c^g}{1-a^g}$.

AG2018: $c^g = 0$.

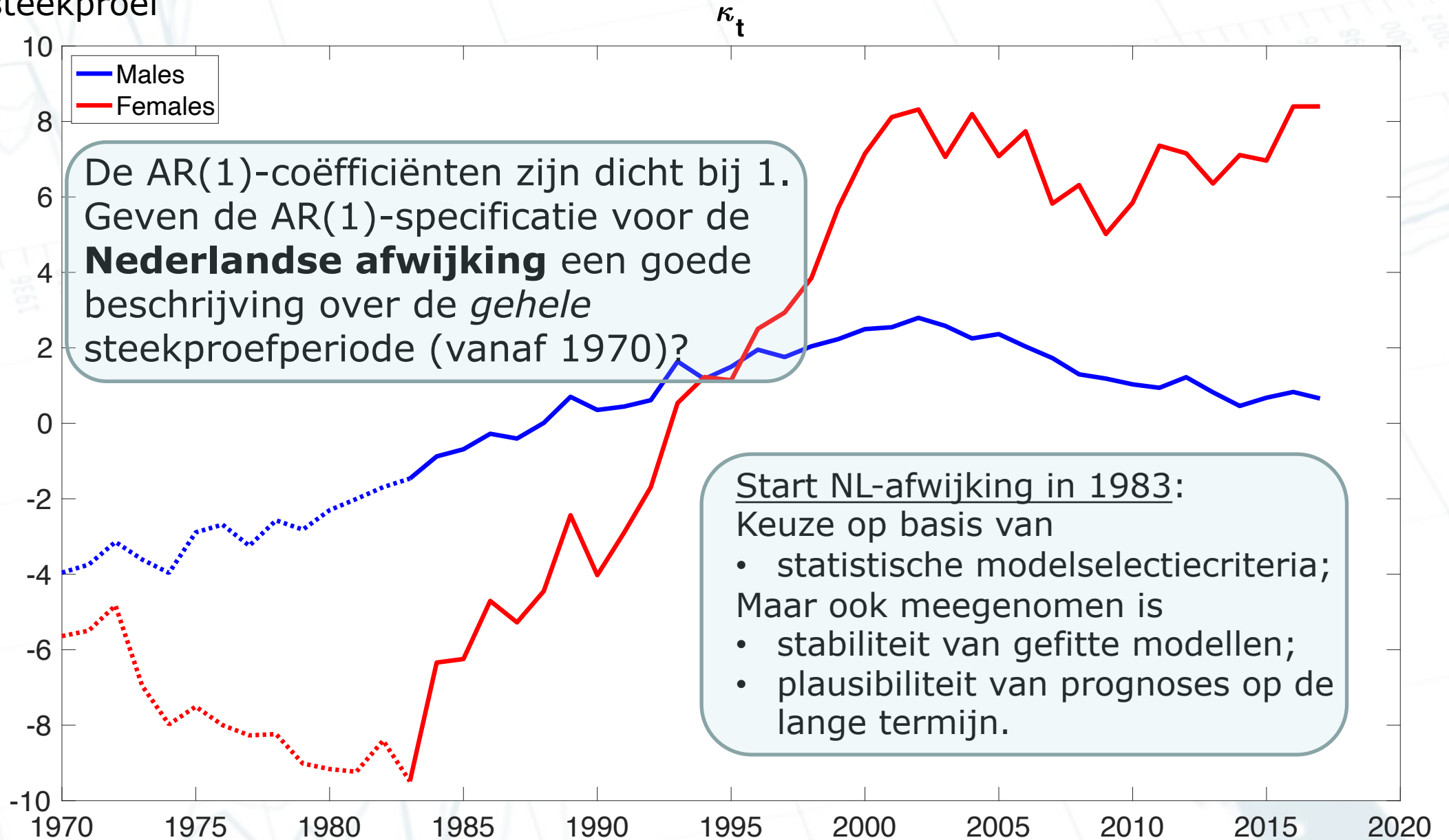
AG2020: c^g ook geschat.

Lengte steekproef

K_t



Lengte steekproef





AG2020



AG2020-Model:

Kalibratie Europa

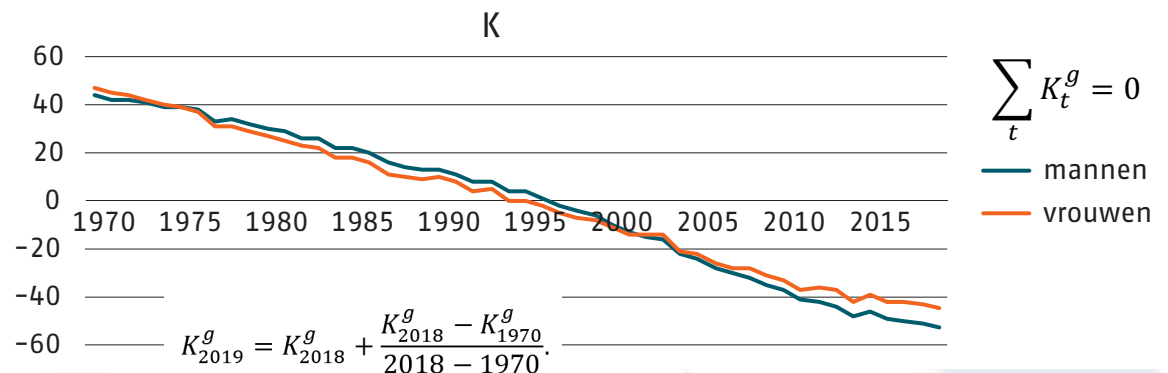
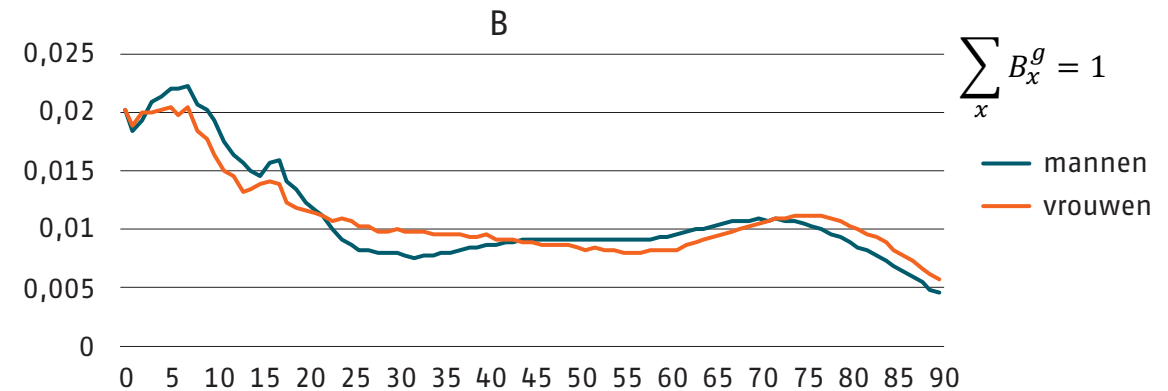
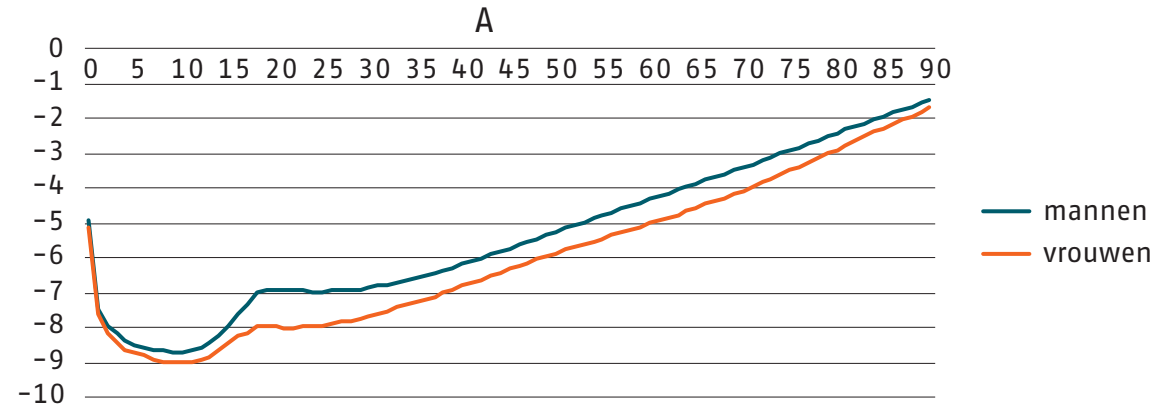
$$D_{x,t}^{g,EU} | E_{x,t}^{g,EU} \sim \text{Pois}(\mu_x^{g,EU}(t) E_{x,t}^{g,EU})$$

Maximum Likelihood:

$$\max_{\{A_x^g, B_x^g, K_t^g\}} \prod_{x \in X^0} \prod_{t \in T^0} \frac{(E_{x,t}^{g,EU} \mu_x^{g,EU}(t))^{D_{x,t}^{g,EU}} \exp(-E_{x,t}^{g,EU} \mu_x^{g,EU}(t))}{D_{x,t}^{g,EU}!},$$

$$\mu_x^{g,EU}(t) = e^{A_x^g + B_x^g K_t^g}$$

$$X^0 = \{0, 1, 2, \dots, 90\}, T^0 = \{1970, \dots, 2018\}$$



AG2020-Model:

Kalibratie NL-afwijking

$$D_{x,t}^{g,NL} | E_{x,t}^{g,NL} \sim \text{Pois}(\mu_x^g(t) E_{x,t}^{g,NL})$$

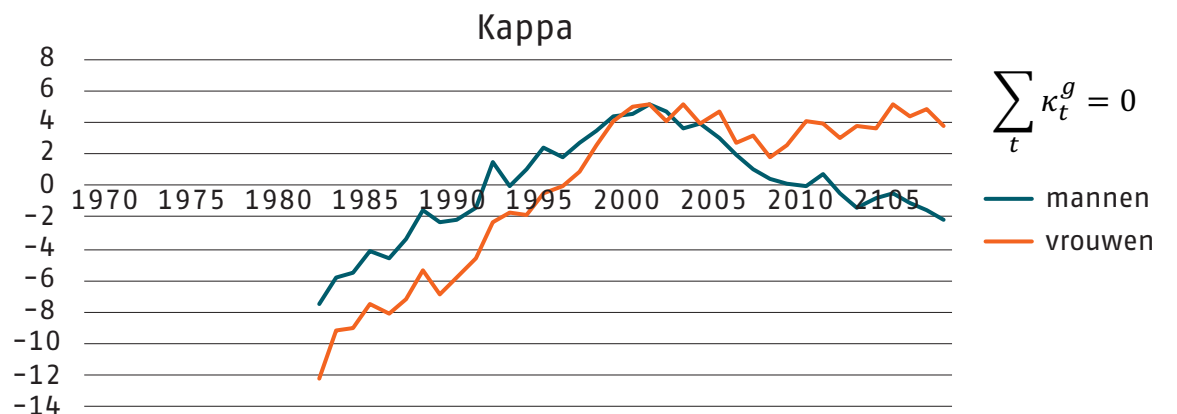
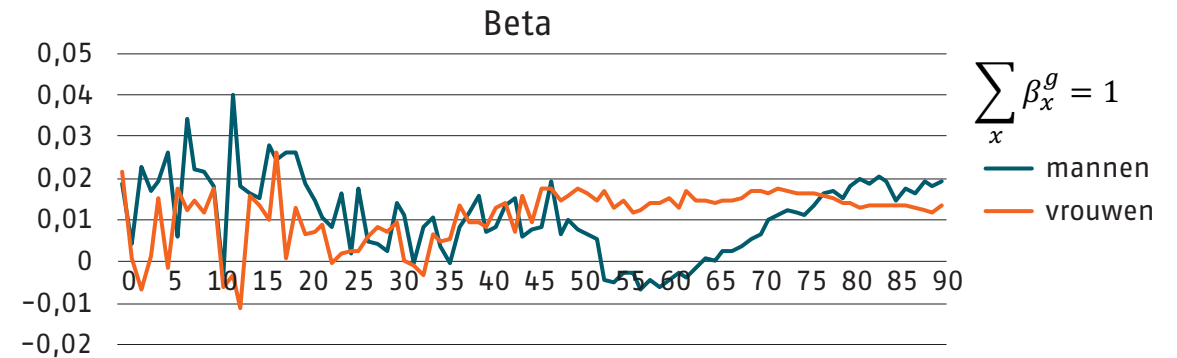
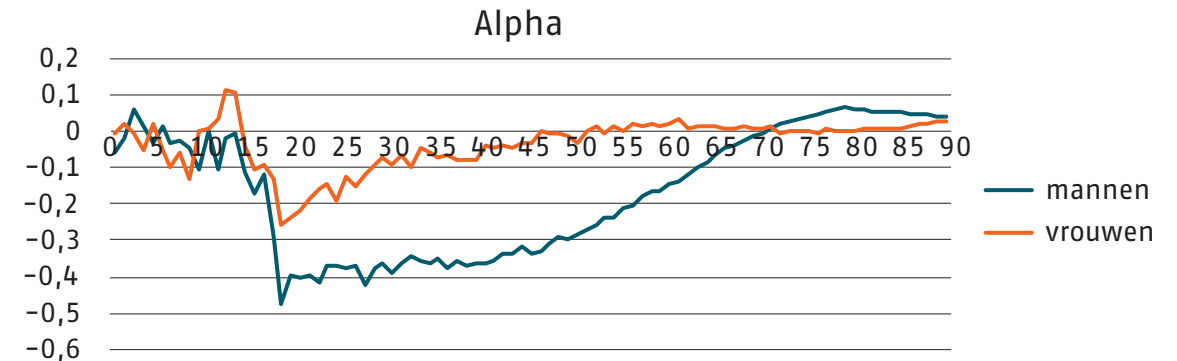
Maximum Likelihood:

$$\max_{\{\alpha_x^g, \beta_x^g, \kappa_t^g\}} \prod_{x \in X^0} \prod_{t \in T^*} \frac{(E_{x,t}^{g,NL} \mu_x^g(t))^{D_{x,t}^{g,NL} \exp} (-E_{x,t}^{g,NL} \mu_x^g(t))}{D_{x,t}^{g,NL}!},$$

$$\mu_x^g(t) = \mu_x^{g,EU}(t) \times \mu_x^{g,NL}(t), \text{ met}$$

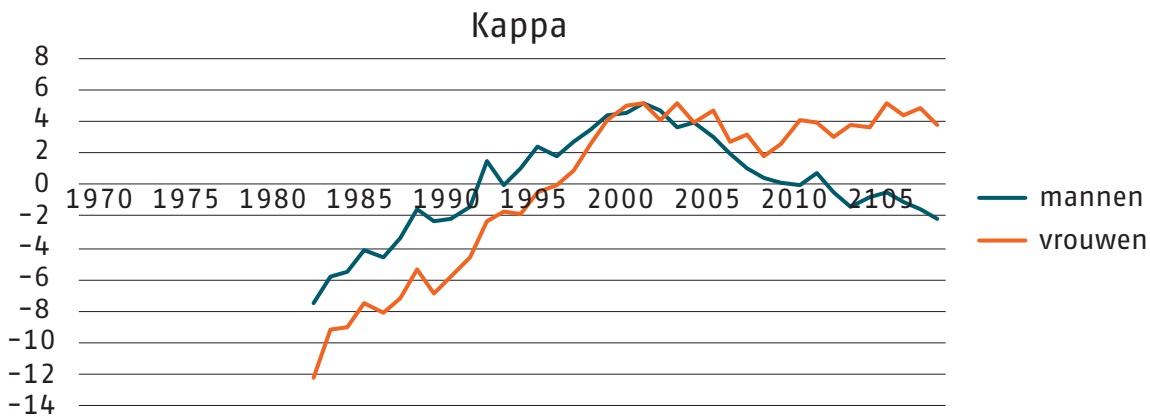
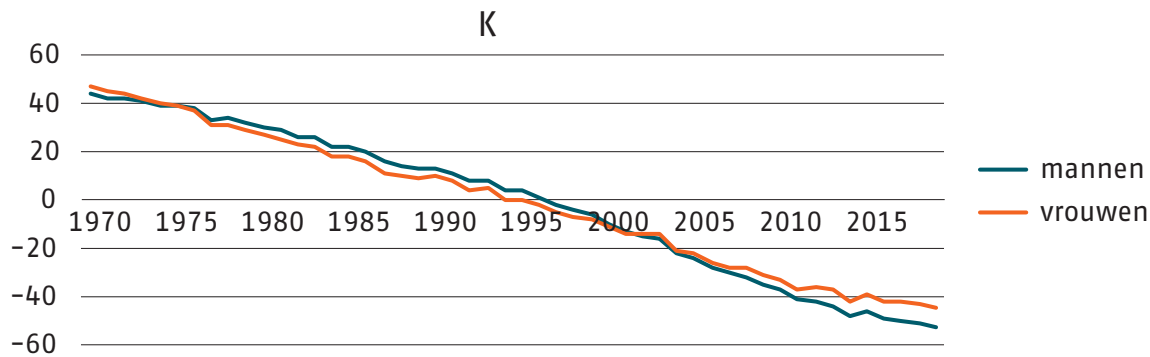
- $\mu_x^{g,EU}(t) \rightarrow \hat{\mu}_x^{g,EU}(t) = e^{\hat{A}_x^g + \hat{B}_x^g \hat{R}_t^g}$
- $\mu_x^{g,NL}(t) = e^{\alpha_x^g + \beta_x^g \kappa_t^g}$

$$X^0 = \{0, 1, 2, \dots, 90\}, T^* = \{1983, \dots, 2019\}$$



AG2020-Model:

Kalibratie tijdstrends:



Modelveronderstelling en Schattingsresultaten:

$$K_t^m = K_{t-1}^m + \theta^m + \epsilon_t^m, \text{ met } \hat{\theta}^m \approx -1.96$$

$$K_t^v = K_{t-1}^v + \theta^v + \epsilon_t^v, \text{ met } \hat{\theta}^v \approx -1.86$$

$$\kappa_t^m = c^m + a^m \kappa_{t-1}^m + \delta_t^m, \hat{a}^m \approx 0.93, \hat{c}^m \approx 0.20$$

$$\kappa_t^v = c^v + a^v \kappa_{t-1}^v + \delta_t^v, \hat{a}^v \approx 0.94, \hat{c}^v \approx 0.41$$

$$(\epsilon_t^m, \epsilon_t^v) \sim i.i.d. N(0, C_{11}), t = 1971, \dots, 1983$$

$$(\epsilon_t^m, \epsilon_t^v, \delta_t^m, \delta_t^v) \sim i.i.d. N(0, C), t = 1984, \dots, 2019$$

$$C = \begin{pmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{pmatrix}$$

AG2020-Model:

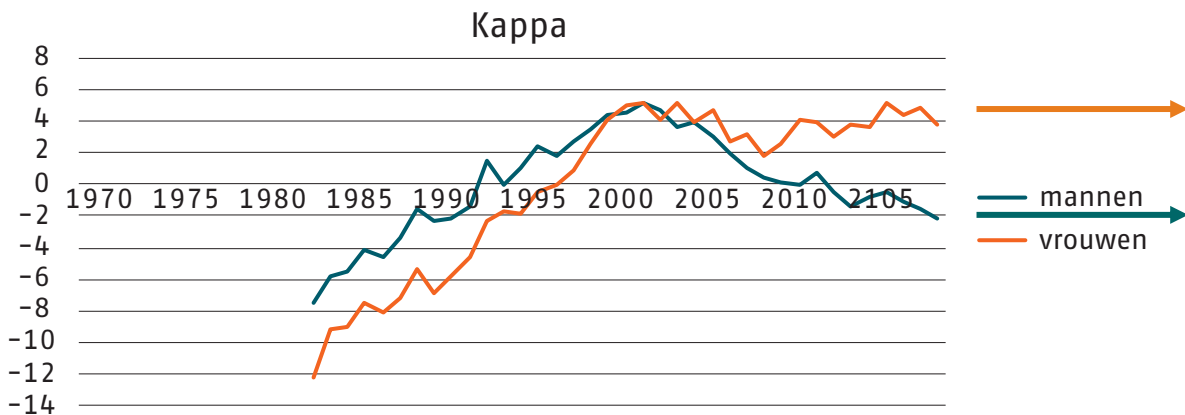
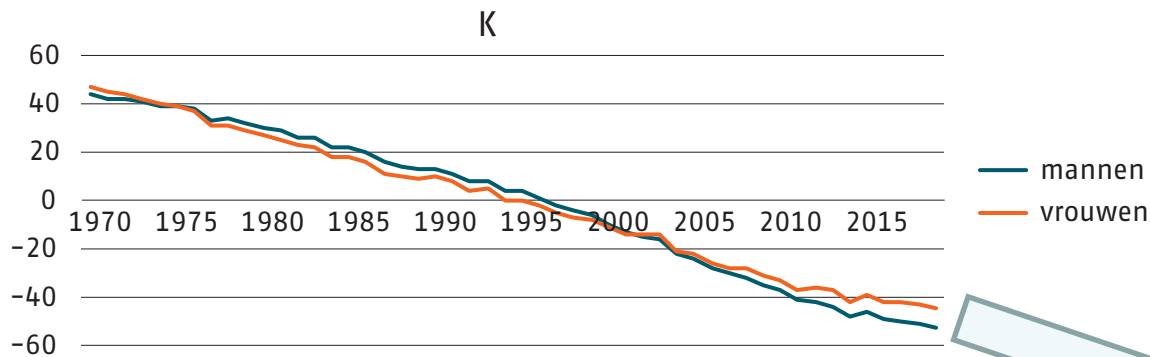
Projecties:

$$q_x^g(T+t) = 1 - e^{-\mu_x^g(T+t)} \text{ met}$$

$$\ln(\mu_x^g(T+t))$$

$$= \ln(\mu_x^{g,EU}(T+t)) + \ln(\mu_x^{g,NL}(T+t))$$

$$= A_x^g + B_x^g K_{T+t}^g + \alpha_x^g + \beta_x^g \kappa_{T+t}^g$$



“Best estimates” K_{T+t}^g

“Best estimates” κ_{T+t}^g

$$\rightarrow \frac{c^v}{1 - a^v} \approx 7,89$$

$$\rightarrow \frac{c^m}{1 - a^m} \approx 2,99$$



$$q_x^g(t) = 1 - e^{-\mu_x^g(t)}$$

Sluiting Prognosetafel

- Leeftijdsafhankelijke parameters voor leeftijden 0 – 90.
- Voor leeftijden 91 – 120 wordt **Kannisto's sluitingsmethode** gebruikt.

Voor leeftijden $x \in \{91, \dots, 120\}$, volgt $\mu_x^g(t)$ uit

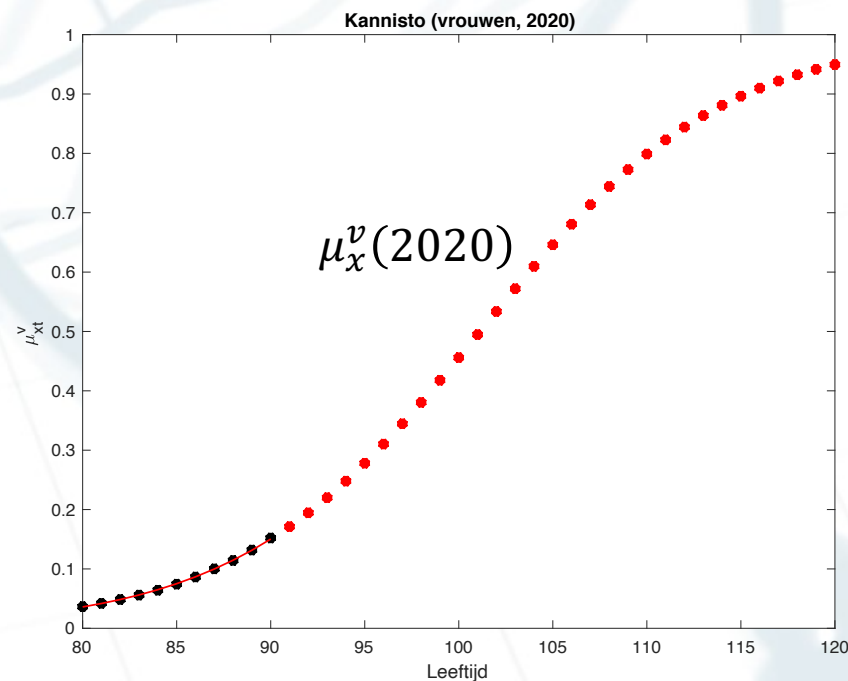
- $$\mu_x^g(t) = L\left(\sum_{k=80}^{90} w_k(x) L^{-1}\left(\mu_k^g(t)\right)\right),$$

met

- $$L(x) = \frac{1}{1+e^{-x}},$$

- $$L^{-1}(x) = -\ln\left(\frac{1}{x}-1\right) = \ln(x) - \ln(1-x),$$

- $$w_k(x) = \frac{1}{11} + \frac{(k-85)(x-85)}{110}.$$



A background image featuring a line chart with multiple data series plotted over time. The x-axis is labeled with years from 1928 to 2004. A magnifying glass is positioned over the chart, focusing on the data points around the year 1970. The chart lines are light blue and grey, and the magnifying glass is a semi-transparent light blue.

AG2020 in vergelijking met AG2018

AG2020 ↔ AG2018



AG2020: Meer “stabiliteit”

- AG2020:

$$\kappa_t^m = c^m + a^m \kappa_{t-1}^m + \delta_t^m, \hat{a}^m \approx 0.93, \hat{c}^m \approx 0.20$$

$$\kappa_t^v = c^v + a^v \kappa_{t-1}^v + \delta_t^v, \hat{a}^v \approx 0.94, \hat{c}^v \approx 0.41$$

- AG2018:

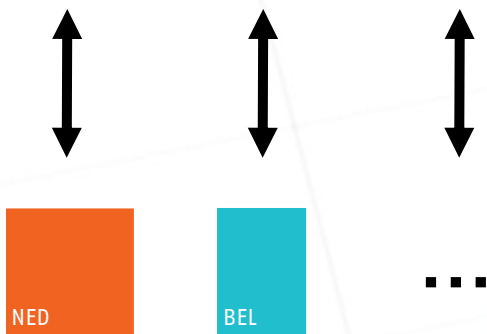
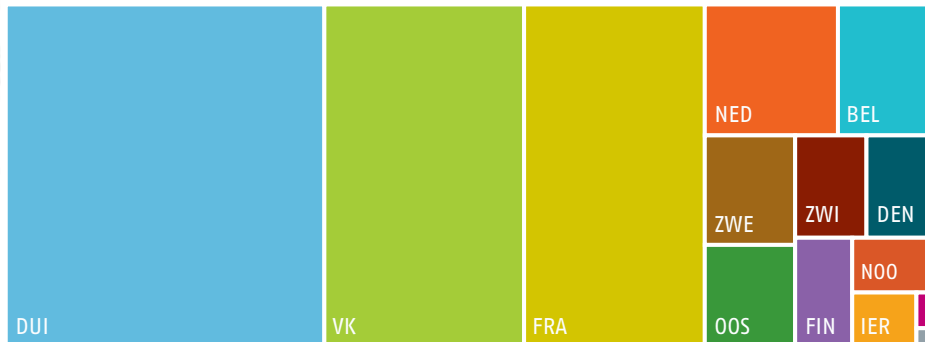
$$\kappa_t^m = a^m \kappa_{t-1}^m + \delta_t^m, \text{ met } \hat{a}^m \approx 0.98$$

$$\kappa_t^v = a^v \kappa_{t-1}^v + \delta_t^v, \text{ met } \hat{a}^v \approx 0.99$$

AG2018: De AR(1)-coëfficiënten zijn dichtbij 1 (instabiliteit).

AG2020: De AR(1)-coëfficiënten zijn verder verwijderd van 1 (meer stabiliteit).

AG2020: Europa-consistent



Europa: “Random walk with drift” (zelfde steekproef)

& correlaties

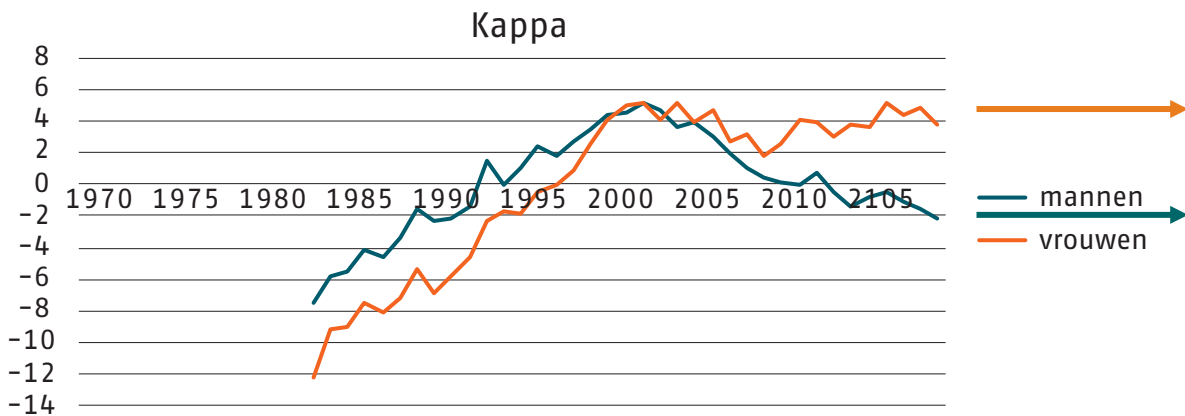
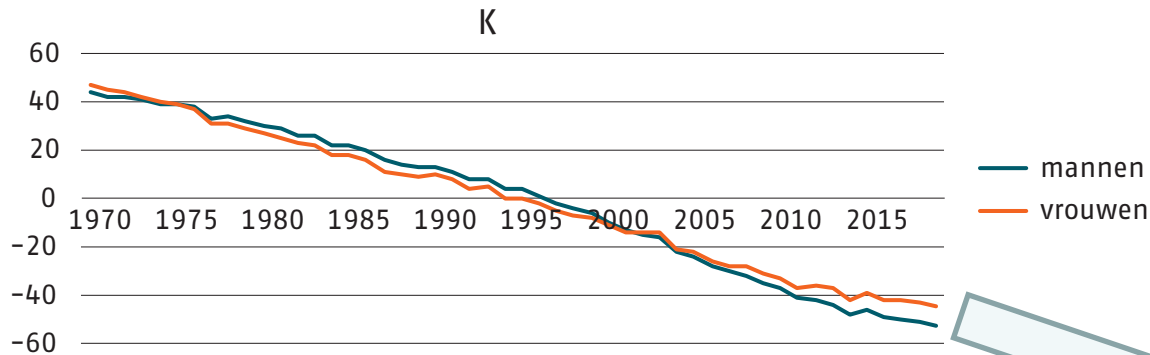
Afwijking: AR(1) met constante.

$$\ln(\mu_x^g(t)) = A_x^g + B_x^g K_t^g + \alpha_x^g + \beta_x^g \kappa_t^g,$$

met

- $K_t^g = K_{t-1}^g + \theta^g + \epsilon_t^g$ **niet** landspecifiek;
- $\kappa_t^g = c^g + a^g \kappa_{t-1}^g + \delta_t^g$ wel landspecifiek.

AG2020: Tijdsconsistent



Tijdsconsistentie:
 Toekomstige realisaties gelijk aan "best estimates"
 \Rightarrow
 Gelijkblijvende parameterschattingen
 (bij gelijkblijvende covariantiematrix C).

"Best estimates" K_{T+t}^g

"Best estimates" κ_{T+t}^g

$$\rightarrow \frac{c^v}{1 - a^v} \approx 6,83$$

$$\rightarrow \frac{c^m}{1 - a^m} \approx 2,86$$

Afwijking NL-EU:

- Afwijking is $\alpha_x^g + \beta_x^g \kappa_{T+\tau}^g \rightarrow_{\tau \rightarrow \infty} \alpha_x^g + \beta_x^g \kappa_{\infty}^g$.

- AG2018: $\alpha_x^g + \beta_x^g \kappa_{\infty}^g = \alpha_x^g$.

- AG2020: $\alpha_x^g + \beta_x^g \kappa_{\infty}^g = \alpha_x^g + \beta_x^g \frac{c^g}{1-a^g}$.

- Voor hogere leeftijden:

- Limietafwijking AG2020 groter dan die van AG2018;
- Gevolgen voor levensverwachting op kortere termijn.

- Afwijking speelt steeds minder belangrijke rol vanwege steeds verder dalende Europese trend.

- Levensverwachtingen NL en EU op langere termijn steeds dichterbij elkaar.

