BY BRAM JOCHEMS
AND ALEXANDER VAN HAASTRECHT

Prepayment modelling with Replication Portfolios

Mortgages are an important asset on the balance sheet for all large Dutch financial institutions. Within mortgages embedded interest rate optionality exists as mortgagors have the possibility to prepay part of their mortgage without penalty or break fees each year or prepay in full when moving to another house. This exercise behaviour is dependent on interest rates and therefore the prepayment option can be seen as interest option at the side of the client. Suitable management of the prepayment behaviour, requires frequent revaluation of mortgage portfolios which can be done with replicating portfolios.

BACKGROUND

Historically, Dutch financial institutions have been applying static prepayment models, that specify a fixed (term structure of) prepayment rate(s) to be applied to the outstanding mortgage portfolio. Doing so underestimates the risks this option poses as it ignores the losses related to the non-linear behaviour of this optionality.

Due to regulatory requirements and advances in computation and modelling capabilities, there is an increasing pressure on financial institutions to use more sophisticated prepayment models. In this article we describe how the risk management of prepayment optionality can be aided by using replicating portfolios.

THE IMPACT OF PREPAYMENT ON MORTGAGE PORTFOLIO VALUATION

Due to their attractive risk-return profile, Dutch banks and insurers have a significant amount of mortgages on their balance sheet. Figure 1 shows outstanding home mortgage lending broken down by sector.

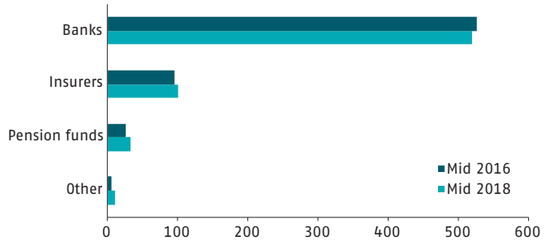

Figure 1: Outstanding home mortgage lending by sector
(Source: DNB)

In the Netherlands, typical conditions for retail mortgages allow mortgage holders to prepay penalty free in the case of certain 'life events' (such as moving to another house) and/or to prepay yearly 10%-20% of their original outstanding amount without any penalty payments. For this reason, an important factor for the valuation, risk and capital calculations associated with mortgages, is what prepayment model and/or assumptions are applied.

To illustrate the impact of prepayment rates, figure 2 shows how the present value of two typical mortgages, one without any scheduled notional repayments and one with annuity payments, varies as a function of a static prepayment rate.

B. Jochems MSc (left) is a partner at Risk at Work and is an expert in managing complex financial risks. At Risk at Work he advises banks and insurers on ALM, hedging and risk management.

Dr. A. van Haastrecht is a partner at Risk at Work and university lecturer in risk management at VU University Amsterdam. He advises banks and insurers in the field of ALM, hedging and risk management.

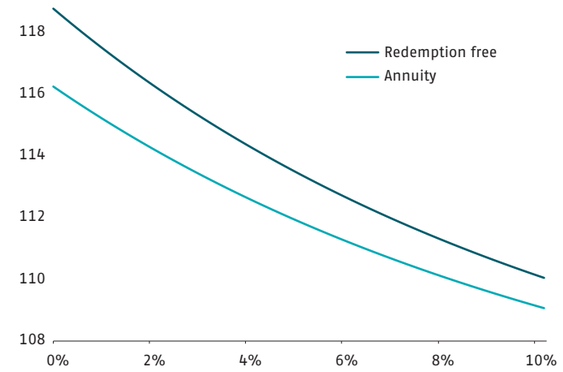

Figure 2: PV of a mortgage as function of a static prepayment rate

Figure 2 established the relationship between the value of a mortgage and a static prepayment rate. Empirical analysis of prepayments shows that prepayments are typically dependent on interest rates; stylistically the relationship is typically similar to figure 3. In this figure a prepayment rate is shown on the y-axis that is a function of refinancing (mortgage) rates on the x-axis. Although the values in the graph will depend on the current and historic economic and interest rate environment, it's typical for prepayment rates to be a non-linear decreasing function of interest (or mortgage) rates. This is easily understood from the observation that as refinancing becomes more attractive, a larger incentive will evoke more actions from mortgagors.

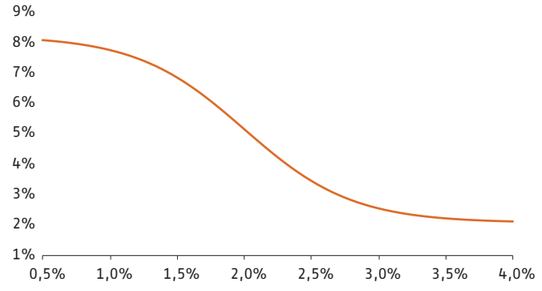

Figure 3: typical behavior for prepayment rate as a function of refinance rates

Together, figure 2 and 3 establish a non-linear relationship between the economic value of a mortgage and interest rates.

Historically, Dutch financial institutions have been applying static prepayment models, that specify a fixed (term structure of) prepayment rate(s) to be applied to the outstanding mortgage portfolio. Due to regulatory requirements and advances in computation and modelling capabilities, there is an increasing pressure on financial institutions to use more sophisticated prepayment models.

REPLICATING PORTFOLIOS

A prepayment option is a complex, exotic option and analytical valuation of a prepayment option is not possible. In addition, daily valuation using monte carlo methods is typically not feasible in ALM vendor systems for realistic mortgage portfolios.

One solution[1] to the problem of daily monte carlo valuations being still infeasible for most financial institutions, is provided by replicating portfolios. Replicating portfolios are a technique already widely employed by Dutch insurers for other parts of their balance sheets. The concept behind replicating portfolios is to find a portfolio of vanilla financial instruments (e.g. swaps, bonds, or swaptions) that have the same value and risk profile as the underlying portfolio they replicate. If such a portfolio can be found and fits the underlying portfolio sufficiently close, this portfolio can be used for valuation, hedging and/or capital purposes. Finding such a replicating portfolio will still require monte carlo simulations. However, these can typically be done once a month or once a quarter and then on a daily basis, the replicating portfolio can be re-evaluated analytically, which is much faster.

To find a replicating portfolio users need to specify three input components. The first of these are scenarios that describe different economic starting points used for generating stochastic paths. The second input are concrete cash flow models that convert paths for the given scenarios into cash flows. The third input component is a specification of the instruments that are allowed in the replication. Based on these inputs, finding a replicating portfolio can then be formulated as a mathematical optimization problem. This is schematically depicted in figure 4.

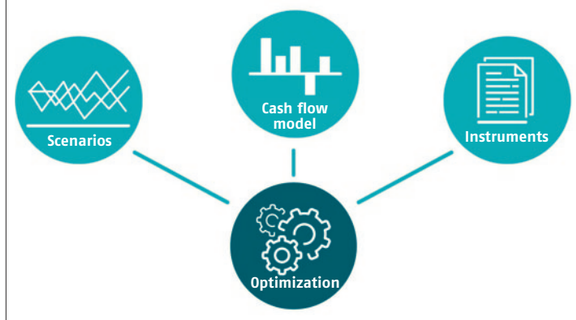

Figure 4: Graphic depiction of Replication methodology

CASE STUDY

To illustrate the preceding discussion, we'll now provide a case study based on representative mortgage portfolio of EUR 1bln, consisting of mortgages typical for the Dutch mortgage market. Figure 5 shows some characteristics of the portfolio.

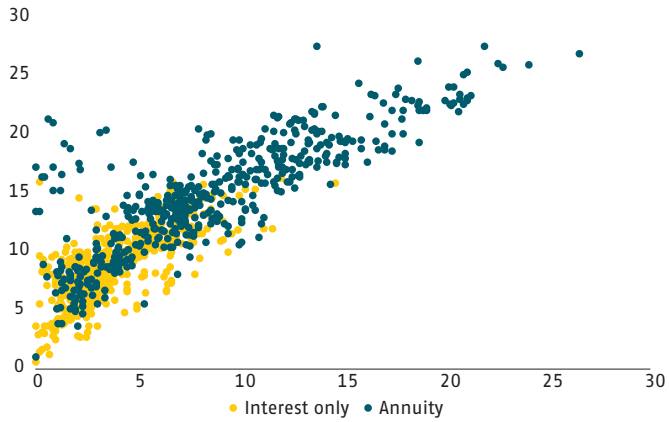

Figure 5a: distribution of time to repricing (x-axis) versus time to maturity (y-axis) for mortgage portfolio used

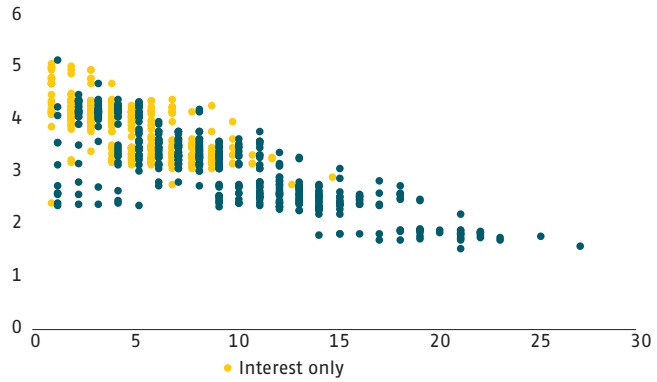

Figure 5b: distribution of coupon rates (y-axis) versus time to reprice for the mortgage portfolio used

For the sake of this case study, we will apply 3 scenarios, consisting of a base scenario with a flat interest rate term structure of 50 basis points and two shocked scenarios that represent a parallel shock of -10 and -50 basis points respectively.

In order to focus on the prepayment optionality, the cash flows obtained from applying a 0% prepayment rate to the portfolio are subtracted from the actual cash flows including dynamic prepayments. This way, really just the prepayment option is replicated and not the interest rate profile related to the discounting of the mortgage portfolio.

The replicating portfolio is then found by fitting a set of zero coupon bonds and receiver swaptions on the cash flows, by fitting on the NPVs per path and adding a penalty for greek mismatches.

RESULTS

The results of the simulation are shown in table 1 and figure 6. The figure shows a scatter plot of the NPV of the mortgage portfolio versus the NPV of the replicating portfolio, per path. Overlaid on the figure is the line y=x. In case of a perfectly replicating portfolio, all points would lie exactly on this line. As the figure indicates, a very good fit was obtained for this sample problem.

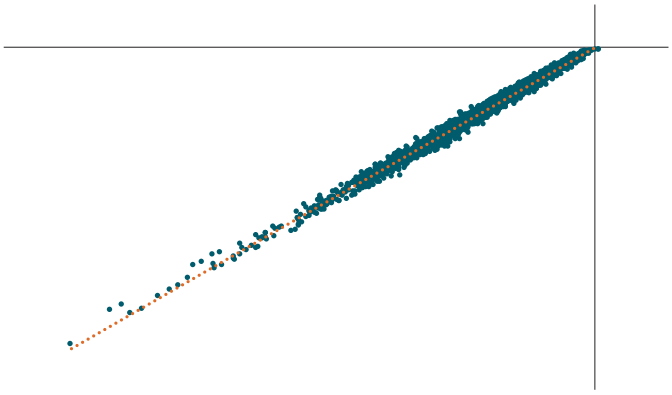

Figure 6: Mortgage NPV vs Replicating portfolio NPV per path

The good fit is also demonstrated by table 1, which shows some quality-of-fit statistics. In addition to the per-path fit looking good (as evidenced by e.g. the high R2), we also note that the change in value between the base scenario and the other scenarios are aligned. In particular the sensitivity towards a 10 basis point down shock shows that the delta of the replicating portfolio is close to that of the mortgage portfolio.

	NPV		Absolute sensitivity				
Scenario	Product	RP	Product	RP	Sensitivity error	R^2	RMSE
Base	-11,650,836	-11,663,643				99.0%	782,120
-10bps	-12,728,659	-12,734,295	-1,077,822	-1,070,652	-0.70%	99.1%	806,186
-50bps	-17,571,965	-17,573,442	-5,921,129	-5,909,799	-0.20%	99.2%	923,102

Table 1: Quality-of-fit metrics for replicating portfolio

Another interesting insight from table 1 can be obtained by subtracting 5 times the sensitivity of the 10 basis point shock from sensitivity of the 50 bps down shock. This provides an indication of the instantaneous losses that would accrue due to optionality in case a perfect linear hedge was in place and a large shock occurred. Extrapolating that number to a 200 basis point shock as a proxy for a 1-in-200 shock for capital calculations, leads to a 2mm EUR loss on a reference portfolio of EUR 1bln, indicating that this is risk is material.

SUMMARY

In this article, we have explained how that the prepayment option embedded in mortgages can be seen as an interest rate option. Including the dynamic prepayment option leads to an improved risk management practice as the risks related to the non-linear prepayment behaviour are explicitly taken into account and can be managed accordingly.

However, valuation of this option, is typically very time-consuming; for a representative portfolio we have demonstrated that a high-quality replicating portfolio can be found to specifically match prepayment optionality risks. Using the replicating portfolio, an efficient revaluation for risk management becomes feasible.

One of the tools for improving the risk management and hedging of mortgage prepayment risks can be hence be to make us of replicating portfolios. ■

1 – Other alternatives exists, e.g. building tailor made distributed monte carlo systems or doing curve fitting