BY JOHN H.J. EINMAHL

We consider the fundamental question whether the probability distribution of the human lifespan has a finite endpoint or not and, if so, whether this upper limit changes over time and of course also how to estimate it. Crucially, this limit is not defined as the highest observed age at death but the highest age that possibly could be reached.

Can we possibly live to 1000 years?

Prof. dr. J.H.J. Einmahl is Arie Kapteyn professor of Statistics at the Department of Econometrics & OR at Tilburg University.

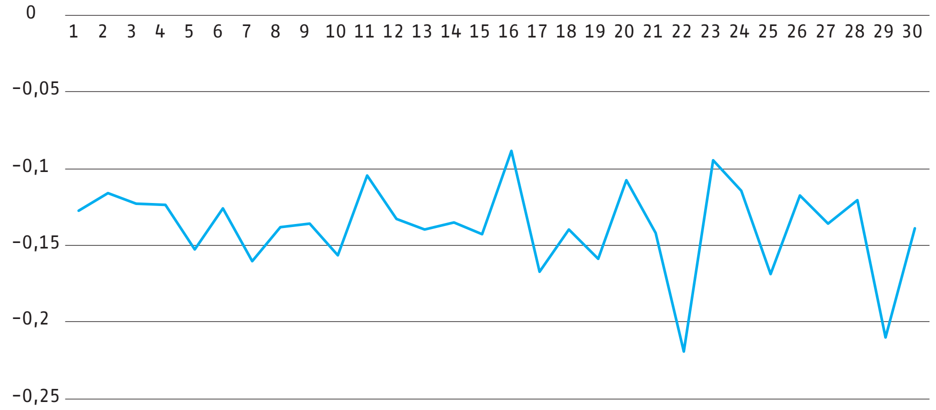
In order to find an answer to the above questions we use a unique dataset (provided by the CBS) of the ages at death in days of all (about 285,000) Dutch residents, born in the Netherlands, who died in the years 1986–2015 at an age of at least 92 years. To reduce bias, we will, however, use only the higher 75,000 of these life spans for our research, which increases the minimum age in the data to 94.4 years. The oldest person in these 30 years, Hendrikje van Andel (a woman), died in 2005 and reached 115.2 years. Remarkably, already more than 100 years before, Geert Adriaans Boomgaard died in 1899 at the age of 110.4 years.

We will not consider all people who were born in a given year but rather all people who died in a given year. In this way we can compare recent years instead of birth years from long ago. We see the people who died in such a given year as a random sample from the imaginary population of all people who could have died in that year. As usual, we will consider women and men separately. Then the goal of our research is to investigate if the life spans of Dutch women and those of Dutch men – per year of death – have an upper limit or not and, if so, to find out if there is some trend in these limits over the period 1986–2015. We will mainly focus on the results for women and then at the end briefly compare with the results for men. This article is based on Einmahl, Einmahl and de Haan (2019), where the reader is referred to for a more comprehensive presentation and for references to related research.

We approach the problem from a statistical point of view using not only the oldest old but the ages at death of the oldest 1500 women per year. Such a large subsample size leads to rather precise statistical inference. The proper statistical context for studying a probability distribution near and beyond the highest observations is Extreme Value Theory (EVT). For a thorough introduction to EVT, see de Haan and Ferreira (2006). EVT does not impose a parametric model; only a weak smoothness condition on the tail of the distribution is assumed. Actually, all continuous, parametric textbook models satisfy this smoothness condition. Note that parametric models typically already assume beforehand, that is without using the data, that the human life span has no upper limit or alternatively that it does have such a limit. This is not the case for the present EVT analysis.

The EVT condition leads to a general model with a so-called extreme value index γ governing the shape of the right tail of the probability distribution. Since the condition only deals with the tail of the distribution it is important that the subsample size (1500), in addition to being large, is relatively small when compared to the total number of Dutch women that died in a given year. This is indeed the case: it is about 2%. Figure 1 shows the estimates of γ for each of the 30 years we consider.

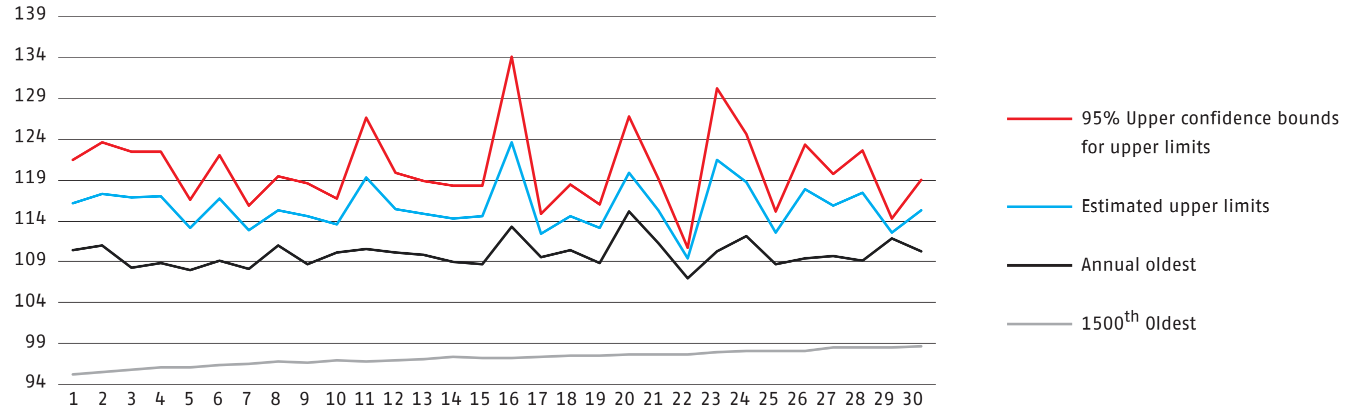
Figure 1 Estimated extreme value indices – women



A special, useful feature of EVT is that the existence of an upper limit of the probability distribution depends on the value of γ. The endpoint is finite if γ is negative and infinite if γ is positive. Obviously all the estimated extreme value indices are negative, but beforehand we wanted to establish that all the true indices γ_j are negative. We perform a formal (simultaneous) test with alternative hypothesis that all the $\gamma_j < 0$. The test is based on the maximum of the estimated extreme value indices. (Here this is the estimated value corresponding to the year 2001 and it is equal to −0.089.) The test leads to an asymptotic *p*-value of 0.0003 (= $\Phi(-0.089\sqrt{1500})$ with Φ the standard normal distribution function) establishing that the upper limit of the probability distribution of the human lifespan of Dutch women is finite for all of the 30 years under consideration.

Once we know that γ is negative, EVT provides a reliable estimate of the upper limit of the probability distribution of the life span. Obviously estimating the upper limit of a probability distribution is an extrapolation technique and hence more difficult and uncertain than estimating a distribution center, like the mean or the median. Figure 2 shows the estimated upper limits (blue) for each of the 30 years. The average estimated upper limit is 115.7 years and the largest estimated upper limit, for the year 2001, is 123.7 years. Using a classical linear or quadratic regression analysis (or a visual inspection) it follows that there is no significant trend in any direction in these estimates. The upper curve (red) provides, taking into account the estimation uncertainty, annual 95% upper confidence bounds for the upper limits of the life span of women. The average of the upper confidence bounds is 120.3 years. In addition, the two lower curves depict respectively the annual oldest (black) and the 1500^{th} oldest (grey) woman. Compared to the 1500th oldests, the annual oldests correctly play only a modest role in our procedure because of their volatility. In contrast to the estimated limits of the life span and the annual oldests (average is 110.0), the ages of the 1500th oldest gradually increase from 95.3 to 98.7 years, an increase of about 0.11 year per calendar year. In other words, we see that the distribution at high age shifts towards the endpoint, over the years, but that the endpoints themselves do not increase.

Figure 2 Estimated upper limits of life span – women



The results for men are qualitatively similar. The test on the extreme value indices γ_j rejects again, i.e., all the endpoints are finite. The estimation results are a bit less precise because in order to guarantee a comparable age threshold we have taken the oldest 1000 men instead of 1500 for women. For men, the age of the 1000^{th} oldest gradually increases from 94.5 to 96.0 years; the average oldest is 107.6. The average endpoint estimate is 114.1 with a maximum estimate of 124.7, with an average 95% upper confidence bound of 119.6. Observe that the average endpoint estimates for women and men are relatively close. Somewhat similarly, we find that for the remaining life time at high age, 100 years say, gender does not play much of a role.

EVT provides also a method to estimate the force of mortality at high age. When measuring time in days, the force of mortality is approximately the probability of dying the coming day. When we consider as high age the "theoretical" time of death of the oldest in a given year, we obtain an average (over the 30 years) estimate for the force of mortality of 0.0031 for women and 0.0029 for men. ■

References

Jesson J. Einmahl, John H.J. Einmahl, and Laurens de Haan (2019). Limits to human life span through extreme value theory, *Journal of the American Statistical Association 114*, 1075–1080.

Laurens de Haan and Ana Ferreira (2006). *Extreme Value Theory, An Introduction* (Springer).